Technical Change and the Labor Share

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AI and the Future of Work

- AI: One of the most important technological breakthroughs in history?
- A lot of angst about the future of work (Will AI take all our jobs?)
- Actually, common theme since Industrial Revolution (e.g., Luddites)
- Can simple economics help us think about this?

Traditional Production Function

- Traditional production function: $Y_t = F(A_t, K_t, L_t)$
- Y_t is output, K_t is capital, L_t is Labor, A_t is "productivity"
- Example: Cobb-Douglas: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$
- Important properties:

1) Returns to scale 2) Elasticity of substitution 3) Labor share

Returns to Scale

Definition: A function f is homogeneous of degree m in x and y if

$$f(\lambda x, \lambda y, z) = \lambda^m f(x, y, z)$$

- $m < 1$: decreasing returns to scale
- $m = 1$: constant returns to scale
- $m > 1$: increasing returns to scale

- Cobb-Douglas:

$$A_t(\lambda K_t)^\alpha(\lambda L_t)^{1-\alpha} = \lambda Y_t$$

Why Constant Returns to Scale?

- We usually assume that the production function is constant returns to scale:

$$F(A_t, \lambda K_t, \lambda L_t) = \lambda F(A_t, K_t, L_t)$$

- Why?

Why Constant Returns to Scale?

- We usually assume that the production function is constant returns to scale:

$$F(A_t, \lambda K_t, \lambda L_t) = \lambda F(A_t, K_t, L_t)$$

- Why?
  - Economy large enough that each establishment has reached efficient size (micro returns to scale and gains from specialization exhausted)
  - Fixed factors (e.g., land) unimportant
  - Positive and negative externalities between establishments unimportant at macro level (cities have achieved efficient size)
  - A_t non-rival (can be used many times)
  - **Replication argument**: Can build a second identical establishment with double the inputs

Euler's Theorem

Euler's Theorem: If f is homogeneous of degree m in x and y:

$$mf(x,y,z) = \frac{\partial}{\partial x} f(x,y,z)x + \frac{\partial}{\partial y} f(x,y,z)y$$

(See Acemoglu (2009, p. 29) for a more careful statement of this theorem.)

- Competitive labor and capital markets:

$$w = \frac{\partial}{\partial L} F(A,L,K) \qquad\qquad r = \frac{\partial}{\partial K} F(A,L,K)$$

- With constant returns production function:

$$Y_t = F(A_t, K_t, L_t) = \frac{\partial}{\partial L} F(A,L,K)L + \frac{\partial}{\partial K} F(A,L,K)K = wL + rK$$

I.e., factor payments exhaust output (no pure profits)

Elasticity of Substitution between Capital and Labor

- Elasticity of substitution between capital and labor is the curvature of the isoquant of the production function
- Slope of an isoquant is the marginal rate of technical substitution:

$$\text{MRTS} = \frac{\partial F(\cdot)/\partial K}{\partial F(\cdot)/\partial L}$$

- Elasticity is then:

$$\left[\frac{d(\text{MRTS})}{d(L/K)} \frac{L/K}{\text{MRTS}}\right]^{-1}$$

- With competitive factor markets: $\text{MRTS} = r/w$
- Elasticity becomes:

$$\frac{d(L/K)}{d(r/w)} \frac{r/w}{L/K} = \frac{d \log(L/K)}{d \log(r/w)}$$

- Percentage change in L/K when r/w changes by one percent
- Intuitively: How easy is it to substitute between labor and capital
- For Cobb-Douglas this equals 1

Traditional Forms of Technical Change

- Hicks Neutral: $A_t F[K_t, L_t]$

  (Ratio of marginal products constant for a given K/L ratio)

- Harrod Neutral / Labor-Augmenting: $F[K_t, A_t L_t]$

  ($F_K K / F_L L$ constant for a given K/Y ratio)

- Solow Neutral / Capital-Augmenting: $F[A_t K_t, L_t]$

  ($F_K K / F_L L$ constant for a given L/Y ratio)

Cobb-Douglas satisfies all three:

- Hicks Neutral:

$$A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Harrod Neutral:

$$K_t^{\alpha} [\tilde{A}_t L_t]^{1-\alpha} \quad \text{where} \quad \tilde{A}_t = A_t^{1/(1-\alpha)}$$

- Solow Neutral:

$$[\check{A}_t K_t]^{\alpha} L_t^{1-\alpha} \quad \text{where} \quad \check{A}_t = A_t^{1/\alpha}$$

Balanced Growth: Kaldor Facts

Kaldor (1963): As per capita income has risen

- The capital-output ratio has been roughly constant
- Real interest rates have no trend
- The labor and capital share of production have been roughly constant

Roughly Constant Capital-Output Ratio

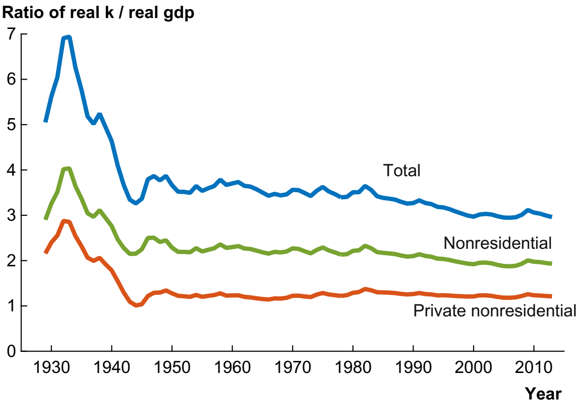
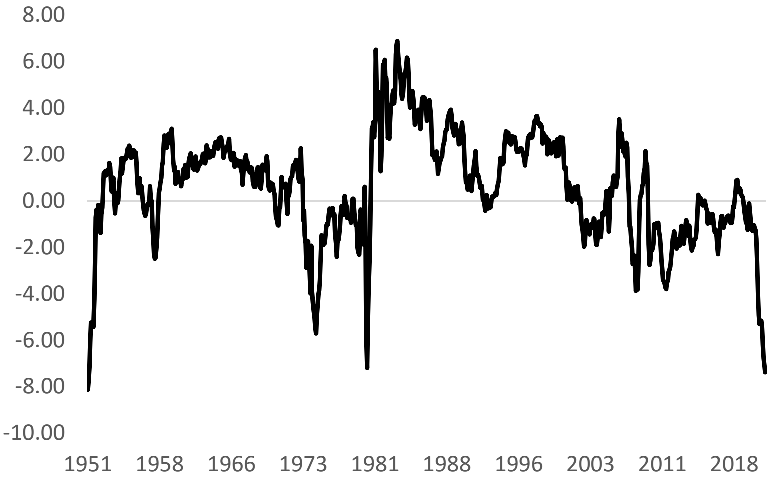


Fig. 3 The ratio of physical capital to GDP. Source: *Burea of Economic Analysis Fixed Assets tables 1.1 and 1.2. The numerator in each case is a different measure of the real stock of physical capital, while the denominator is real GDP.*

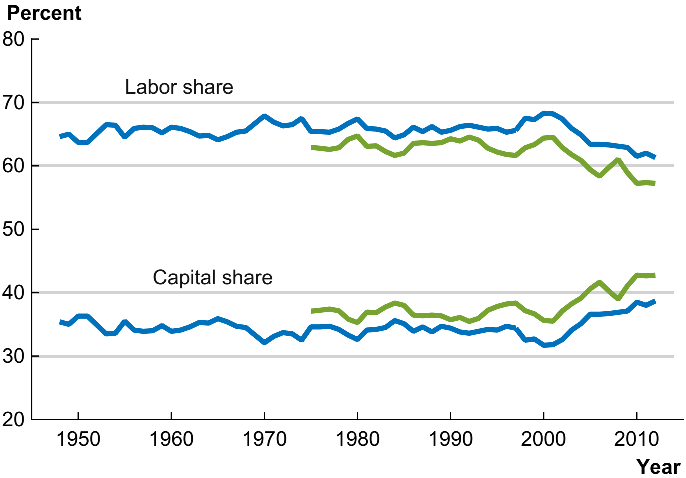
Source: Jones (2016)

Ex Post Real Interest Rate



Source: FRED. 3 month T-bill rate minus 12-month CPI inflation.

Roughly Constant Labor and Capital Shares



Source: Jones (2016)

Uzawa's (1961) Theorem

Roughly speaking:

- Balanced growth in the long run is only possible if all technical progress is labor augmenting

  (See Acemoglu (2009, sec. 2.7) and Barro-Sala-I-Martin (2004, sec. 1.2.12) for details)

Acemoglu (2009, p. 59):

> *This result is very surprising and troubling, since there are no compelling reasons for why technological progress should take this form. [i.e., be labor augmenting]*

Labor Augmenting or Capital Augmenting

- Actually very hard to think about whether any given technological innovation is labor augmenting or capital augmenting
- Is a better computer labor or capital augmenting?
  - Is it like having two old computers? (capital augmenting)
  - Or does it effectively increase the user's time (labor augmenting)
- Not at all clear!!
- Same with most other technological improvements

  (spinning jenny, steam engine, electric motor, tractor, container shipping, AI etc.)
- Task-based production functions will help us think about this

Technology and Labor

- Workers have long worried that labor-saving technology would supplant them
- True ever since onset of Industrial Revolution
- Every technology destroys some jobs. But does it make workers worse off in general?
- Two measures:
  1. Real wages
  2. Labor share of income

Real Wages in the United States

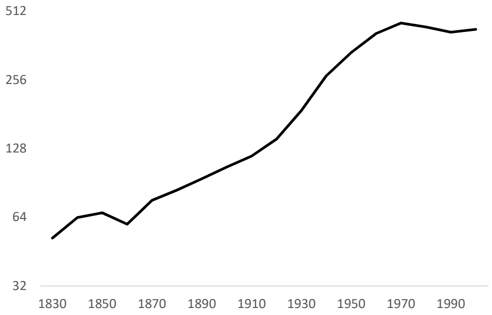


Figure 2: Real Wages in the United States

Note: This figure plots real wages in the United States from 1830 to 2009. Data for the period 1830 to 1988 are from Williamson (1995). Data from 1988 to 2020 is from the Main Economic Indicators of the OECD (average nominal wages in manufacturing). The price series used to deflate nominal wages is the CPI. I plot decadal averages. That is, the data point listed as 1830 is the decadal average for 1830-1839.

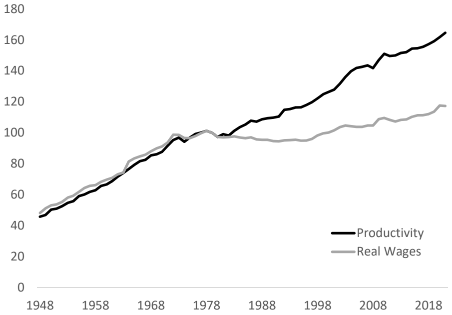


Figure 21: Productivity and Real Wages in the United States

Note: These data series were constructed by the Economic Policy Institute using methodology described in Bivens and Mishel (2015). The black line is productivity measured as output of goods and services less depreciation per hour worked. The gray line is hourly compensation (wages and benefits) of production and non-supervisory workers in the private sector. Data is from 1948 to 2021.

(Deflated by CPI)

Labor Share in the United States

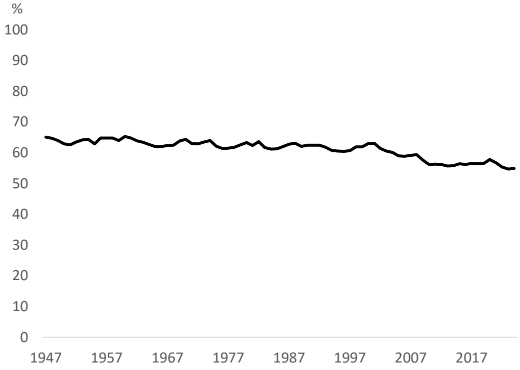


Figure 2: Labor Share in U.S. Non-farm Business Sector

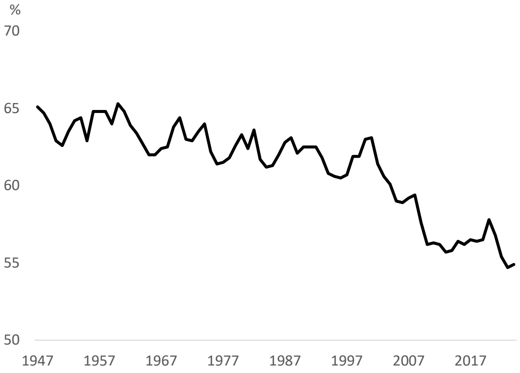


Figure 4: Labor Share in U.S. Nonfarm Business Sector, A Closer Look

Labor Share in the United States

- Fall in labor share goes away if we use pre-1999 methods
- But aren't new methods better?
- They capitalize intel. property products (growing share, high depreciation rate)
- GDP double counts investment
- We are interested in factor shares as measure of ability to consume
- But growing share of GDP goes to maintaining capital stock
- Capital share in NDP better measure

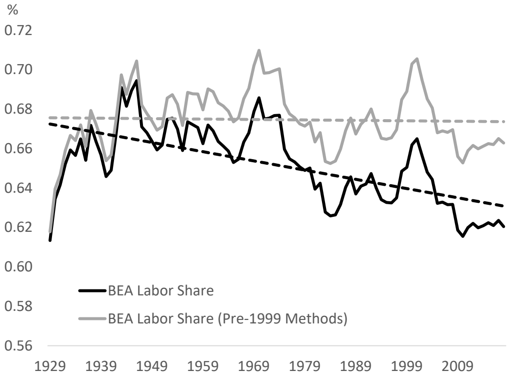


Figure 5: Economy-Wide U.S. Labor Share

(Koh, Santaeulalia-Llopis, and Zheng (2020))

- Smith, Yagan, Zidar, and Zwick (2022): Third of fall in labor share in U.S. corporate sector due to tax-related shifts in how income is reported

- Firms shift from being C corps to being S corps

- High-labor share firms shift towards being partnerships (drop from corporate sector)

(Hard to measure non-corporate sector due to reporting issues)

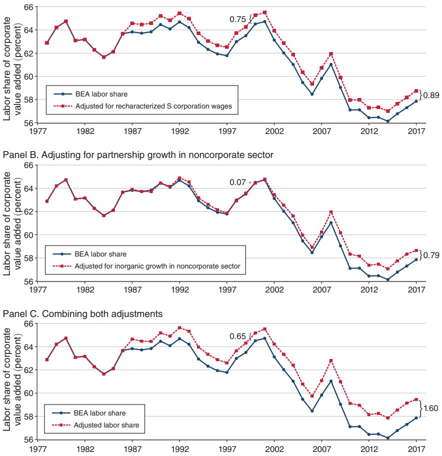


Figure 3. Adjusted Corporate-Sector Labor Shares (1978–2017)

(Smith, Yagan, Zidar, Zwick (2022))

Wages and Labor Share with Cobb-Douglas

- Suppose the production function is Cobb-Douglas $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$
- Wages:

$$w = \frac{\partial Y}{\partial L} = (1-\alpha) A K^{\alpha} L^{-\alpha}$$

- Effect of A on w:

$$\frac{\partial \log w}{\partial \log A} = 1 > 0$$

- Effect of K on w:

$$\frac{\partial \log w}{\partial \log K} = \alpha > 0$$

- Labor share:

$$w = (1-\alpha) A K^{\alpha} L^{-\alpha}$$

$$wL = (1-\alpha) A K^{\alpha} L^{1-\alpha} = (1-\alpha) Y$$

$$\frac{wL}{Y} = (1-\alpha)$$

- So, labor share is constant!
- Labor and capital benefit from technical progress proportionally
- Technical progress does not make workers (overall) worse off if production is Cobb-Douglas

CES Production Function

- Constant labor share for Cobb-Douglas is a knife-edge case
- To see this, we consider a more general class of production functions
- Constant elasticity of substitution (CES):

$$Y_t = \left[\alpha(A_{Kt}K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_{Lt}L_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  - Elasticity of substitution between capital and labor is σ
    - If $\sigma > 1$ labor and capital are "gross substitutes"
    - If $\sigma < 1$ labor and capital are "gross complements"
  - A_{Kt}, A_{Lt} capital and labor augmenting technical change, respectively
- Notice that it is constant returns to scale (this is what outer exponent does)

CES Production Function

- Labor and capital demand:

$$(1-\alpha)A_{L,t}^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} = w_t \qquad \alpha A_{K,t}^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} = r_t$$

- Labor and capital shares:

$$\frac{w_t L_t}{Y_t} = (1-\alpha)\left(\frac{A_{L,t}L_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} \qquad \frac{r_t K_t}{Y_t} = \alpha\left(\frac{A_{K,t}K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}$$

Wages with CES Production

- We can write wages as

$$w_t = (1-\alpha) A_{L,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} = (1-\alpha) A_{L,t}^{\frac{\sigma-1}{\sigma}} \left[\alpha A_{K,t}^{\frac{\sigma-1}{\sigma}} \left(\frac{K_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) A_{L,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}$$

- From this it is easy to see that

$$\frac{\partial w}{\partial K} > 0 \qquad \frac{\partial w}{\partial A_K} > 0$$

- More and higher productive capital benefits workers (true for any value of σ!!)
- This model cannot help us understand how capital-augmenting technical change may hurt workers

Labor Share with CES Production

- Another way to write labor and capital shares:

$$\frac{r_t K_t}{Y_t} = \alpha^\sigma \left(\frac{A_{K,t}}{r_t}\right)^{\sigma-1} \qquad \frac{w_t L_t}{Y_t} = 1 - \alpha^\sigma \left(\frac{A_{K,t}}{r_t}\right)^{\sigma-1}.$$

- If $\sigma > 1$: Increase in $A_{K,t}$ and fall in r_t reduce the labor share
  - Strong substitution towards capital when it gets better or cheaper
- If $\sigma < 1$ it's the opposite
  - Weak substitution towards capital when it gets better or cheaper

Karabarbounis and Neiman (2014)

- Relative price of investment has fallen
- This is like a fall in r_t
- Argue that $\sigma > 1$
- Explains fall in labor share

- But is $\sigma > 1$ really true?
- Many argue $\sigma < 1$
  (e.g., Oberfeld and Raval, 2021)

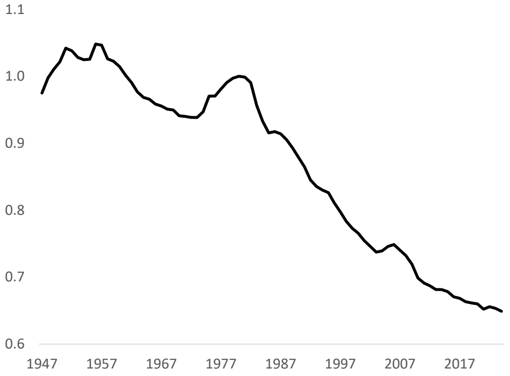


Figure 6: The Price of Investment Goods Relative to the Price of Output

Estimating Capital-Labor Elasticity

- Labor and capital demand:

$$(1-\alpha)A_{L,t}^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} = w_t \qquad \alpha A_{K,t}^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}} = r_t$$

- Taking the ratio of these and then taking logs yields:

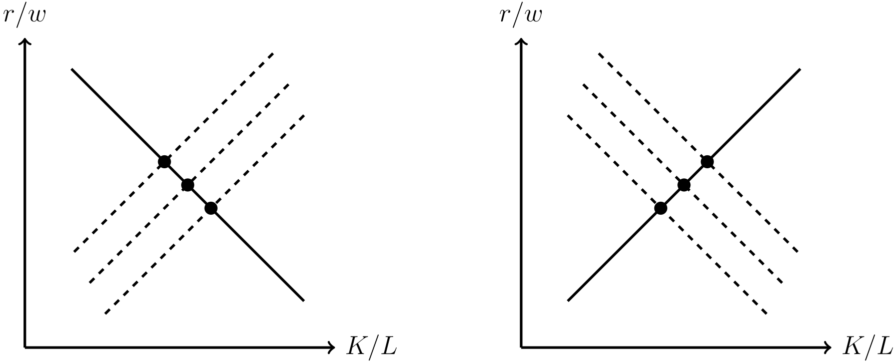
$$\log\left(\frac{K_t}{L_t}\right) = \sigma\log\left(\frac{w_t}{r_t}\right) + \sigma\log\left(\frac{\alpha}{1-\alpha}\right) + (1-\sigma)\log\left(\frac{A_{L,t}}{A_{K,t}}\right)$$

  Or more concisely:

$$\log\left(\frac{K_t}{L_t}\right) = \sigma\log\left(\frac{w_t}{r_t}\right) + \epsilon_t$$

  where ϵ_t is variation in $\log(A_{L,t}/A_{K,t})$

- σ is the slope of the relative factor demand curve



"Good" Variation "Bad" Variation

Figure 7: Estimating Elasticity of Substitution

Estimating Capital-Labor Elasticity

- We want **aggregate** σ. Not the same as firm or industry σ
- When R/W falls (say), each firm and industry will respond
- But substitution will also occur across firms and industries
- Houthakker (1955):
  - Each firm Leontief ($\sigma = 0$)
  - Each firm has fixed production capacity
  - Capacity follows Pareto distribution across technologies
  - Aggregate production function Cobb-Douglas ($\sigma = 1$)

The "Canonical Model" in Labor Economics

- Katz and Murphy (1992), Acemoglu and Autor (2011)
- Two types of workers high and low "skill" (usually means education)
- Production function:

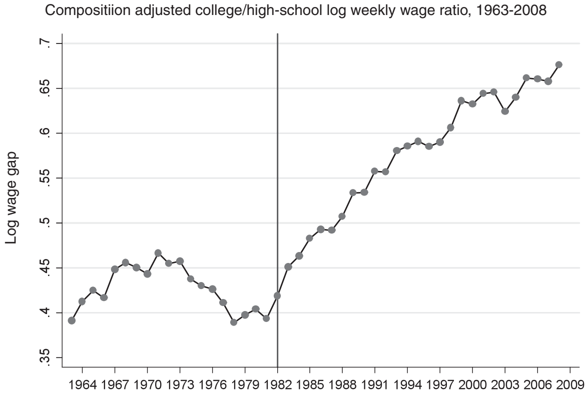
$$Y = \left[(A_L L)^{\frac{\sigma - 1}{\sigma}} + (A_H H)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}$$

- L is quantity of low-skilled labor, H is quantity of high-skilled labor
- σ is the elasticity of substitution between low- and high-skilled labor
- Technology (A_L and A_H) is factor-augmenting (no directly factor replacing tech)

College / High School Skill Premium

- The College / High School wage premium has risen substantially since 1980

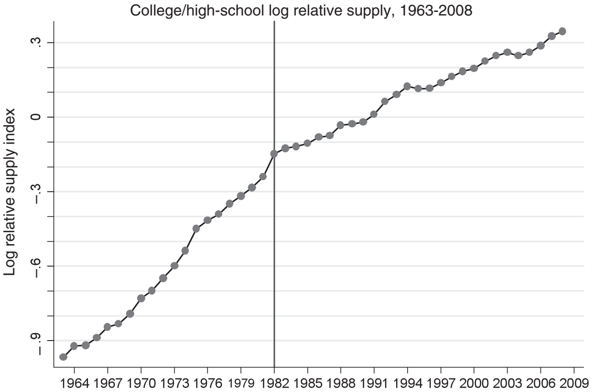
Update



(Acemoglu and Autor (2011))

Relative Supply of High-Skilled Labor

- Relative supply of skilled labor has risen substantially
- Together, these two facts suggest a large shift in relative demand for labor favoring high-skilled labor



(Acemoglu and Autor (2011))

Wages of Low Skilled in the Canonical Model

- Suppose labor market is competitive
- Low-skilled wages:

$$w_L = \frac{\partial Y}{\partial L} = A_L^{\frac{\sigma-1}{\sigma}} \left[A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} (H/L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

- This implies that:

$$\frac{\partial w_L}{\partial H} > 0 \qquad \frac{\partial w_L}{\partial A_H} > 0 \qquad \frac{\partial w_L}{\partial A_L} > 0$$

- Having more high-skilled workers benefit low-skilled workers
- High-skill-biased technical (A_H) change benefits low-skilled workers (as does A_L)
- Only technological regress can explain falling wages

THE SKILL PREMIUM IN THE CANONICAL MODEL

- Taking the ratio of demand for high-skilled and low-skilled labor:

$$\frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}$$

- Taking logs:

$$\log\left(\frac{w_H}{w_L}\right) = \frac{\sigma-1}{\sigma}\log\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma}\log\left(\frac{H}{L}\right)$$

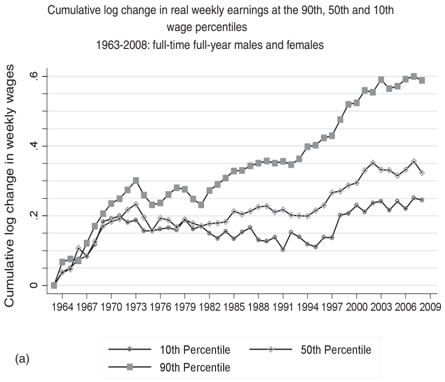
- An increase in H/L reduces the skill premium
- An increase in A_H/A_L increases the skill premium if $\sigma > 1$
  - If $\sigma < 1$ and increase in A_H/A_L increases demand for L more than it increases demand for H. (Think of Leontief case.) It, therefore, decreases the skill-premium. A_H/A_L is skill replacing.
  - Empirical literature suggests $\sigma > 1$ (perhaps 1.4 to 2)

Race Between Education and Technology

- Tinbergen (1974, 1975), Katz and Goldin (2008)

$$\log\left(\frac{w_H}{w_L}\right) = \frac{\sigma-1}{\sigma}\log\left(\frac{A_H}{A_L}\right) - \frac{1}{\sigma}\log\left(\frac{H}{L}\right)$$

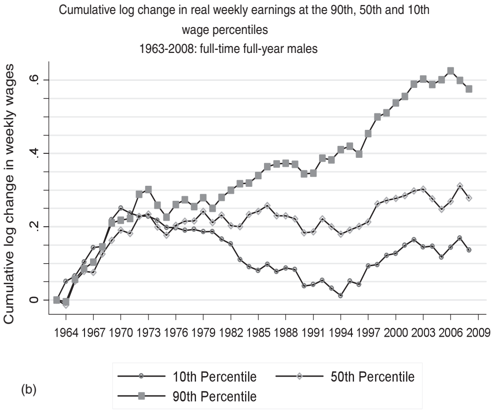
- Skill-biased technical change (increase in A_H/A_L (if $\sigma > 1$)) push up skill premium
- Increases in educational attainment (increase in H/L) push down skill premium
- Rough balance 1950-1980
- Imbalance 1980-2010



(Acemoglu and Autor (2011))

Fall in Low-Skilled Wages for Men?

- By some metrics, wages of low-skilled men fell between 1970 and 1995
- Canonical model cannot generate this
- Fall in wages is somewhat sensitive to various measurement issue (e.g., which price index is used, inclusion of fringe benefits)



(Acemoglu and Autor (2011))

More Flexible Framework?

- CES production function is a powerful framework for many questions
- However, it doesn't capture well the notion that technological change often involves machines replacing workers ... potentially making workers worse off
- Task-based production function models capture this notion
  - Developed by Zeira (1998), Acemoglu and Autor (2011), Acemoglu and Restrepo (2018), Jones and Tonetti (2026) among others

Interlude: Aggregate Production Function?

- All the models we consider in this lecture assume an aggregate production function (except Houthakker (1955))
- But does such a thing exist?
  - What does it mean to talk about aggregate labor, aggregate capital, aggregate output?
- Conditions under which firm production functions can be aggregated are non-trivial
- Huge debate about this in 1950s and 60s ("Cambridge Capital Controversy")
- Two views of simplifying assumptions:
  - They are likely wrong. They sweep part of reality under the rug.
  - They are necessary to allow clear analysis of something more important

Navigating Simplifying Assumptions

- Navigating simplifying assumptions is crucial to being good at economics research
- You need to both know when to make them **and** remember that you are making them
- Some are paralyzed because they are not willing to make any simplifying assumptions
- Some are brainwashed by them and forget the world is more complex than the models
  (This is what Daniel Kahneman called “theory induced blindness”)
- Having good taste / judgment on this front is a crucial skill

Cambridge Capital Controversy

Cambridge, U.K.:

- Harcourt, G.C. (1969): "Some Cambridge Controversies in the Theory of Capital," *Journal of Economic Literature*, 7(2), 369-405.
- Cohen, A.J. and G.C. Harcourt (2003): "Whatever Happened to the Cambridge Capital Theory Controversies?" *Journal of Economic Perspectives*, 17(1), 199-214.

Cambridge, U.S.:

- Samuelson, P.A. (1966): "A Summing Up," *Quarterly Journal of Economics*, 80(4), 568-583.
- Stiglitz, J.E. (1974): "The Cambridge-Cambridge Controversy in the Theory of Capital: A View from New Haven," *Journal of Political Economy*, 82(4), 893-903.

Task-Based Model of Production

- Production of output involves the completion of a continuum of tasks ν
- Production function:

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\sigma-1}{\sigma}} d\nu \right)^{\frac{\sigma}{\sigma-1}}$$

- $Y_t(\nu)$ denotes the output of task ν
- This is a CES production function with a continuum of inputs (the tasks)
- The elasticity of substitution across tasks σ will turn out to be a key parameter

Production Function for Tasks

- The production function for a particular task ν is

$$Y_t(\nu) = \psi_{kt}(\nu)K_t(\nu) + \psi_{\ell t}(\nu)L_t(\nu)$$

- $K_t(\nu)$ and $L_t(\nu)$ are amount of capital and labor used in production of task ν
- $\psi_{kt}(\nu)$ and $\psi_{\ell t}(\nu)$ denote task-specific productivity of capital and labor
- Capital and labor are perfect substitutes in performing task ν

Problem of Final Good Producer

- Final good producer chooses how much of each task to "purchase" to maximizes profits:

$$\max_{[Y_t(\nu)]_{\nu\in[0,1]}} \left(\int_0^1 Y_t(\nu)^{\frac{\sigma-1}{\sigma}} d\nu \right)^{\frac{\sigma}{\sigma-1}} - \int_0^1 P_t(\nu) Y_t(\nu) d\nu,$$

- $P_t(\nu)$ is the price of task ν (and we set the price of the final good to 1 (numeraire))

- First order condition yield demand curve for task ν

$$Y_t(\nu) = P_t(\nu)^{-\sigma} Y_t$$

- Constant elasticity demand curve with elasticity $-\sigma$

Production of Tasks

- Cost of producing task ν with labor: $\frac{W_t}{\psi_{\ell t}(\nu)}$
- Cost of producing task ν with capital: $\frac{R_t}{\psi_{kt}(\nu)}$
- Firm will choose whichever is lower
- Marginal cost of producing task ν

$$MC_t(\nu) = \min\left\{\frac{W_t}{\psi_{\ell t}(\nu)}, \frac{R_t}{\psi_{kt}(\nu)}\right\}$$

- Market for producing tasks is perfectly competitive. So, $P_t(\nu) = MC_t(\nu)$

Which Tasks are Automated

- If $\frac{W_t}{\psi_{\ell t}(\nu)} < \frac{R_t}{\psi_{kt}(\nu)}$, task ν is produced with labor
- If $\frac{W_t}{\psi_{\ell t}(\nu)} > \frac{R_t}{\psi_{kt}(\nu)}$, task ν is produced with capital
- We can order the tasks however we want (they enter production symmetrically)
- We assume that $\frac{\psi_{kt}(\nu)}{\psi_{\ell t}(\nu)}$ is decreasing in ν
- Labor has a comparative advantage at high-index tasks.

Smoothly Decreasing $\psi_{kt}(\nu)/\psi_{\ell t}(\nu)$

- Suppose $\frac{\psi_{kt}(\nu)}{\psi_{\ell t}(\nu)}$ decreases smoothly
- There will be a cutoff task α_t where

$$\frac{R_t}{\psi_{kt}(\alpha_t)} = \frac{W_t}{\psi_{\ell t}(\alpha_t)}$$

- All tasks to the left will be automated
- All tasks to the right produced by labor

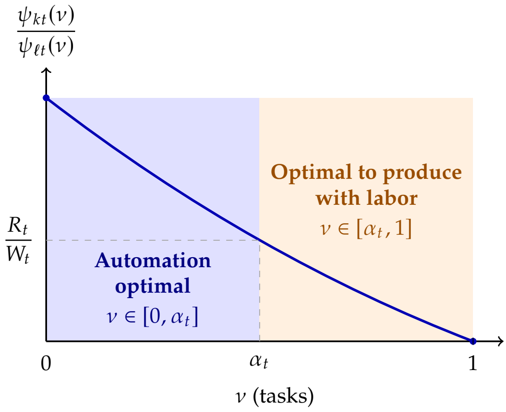


Figure 1. Smooth Automation.

An Alternative View

- Here we plot the cost of production of each task with labor and capital
- Again, all tasks to the left of crossing will be automated
- Technical progress is increases in $\psi_{kt}(\nu)$ and $\psi_{\ell t}(\nu)$
- These shift the lines down (and can shift the crossing point)
- Three categories of innovation

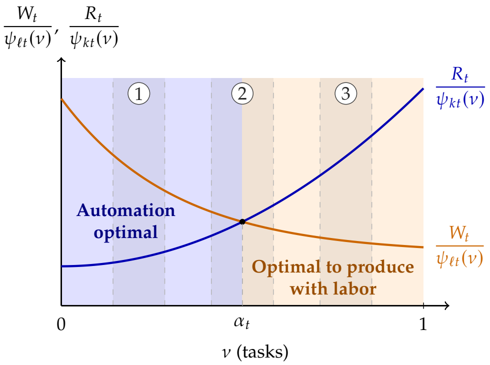


Figure 2. Cost of Producing Tasks with Labor and Capital.

Innovation Deep in Automated Region

- First consider increases in $\psi_{kt}(\nu)$ well to the left of crossing point
- Lowers task prices for these tasks
- No effect on automation
- Pure productivity effect

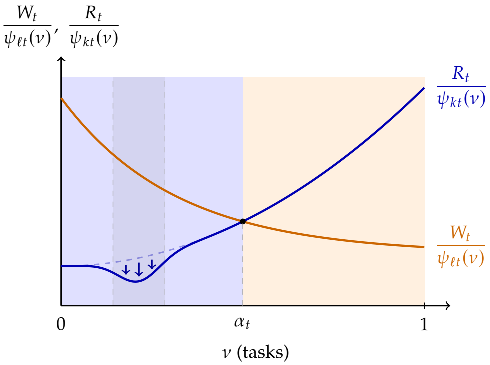


Figure 3. Technical Progress Deep in the Automated Region.

Innovation Near Automation Cutoff

- Next consider increases in $\psi_{kt}(\nu)$ that straddle the crossing point
- Move α_t to the right
- Displace workers from tasks $\nu \in [\alpha, \alpha']$
- Also lowers task prices for these tasks
- Combination of automation and productivity effects

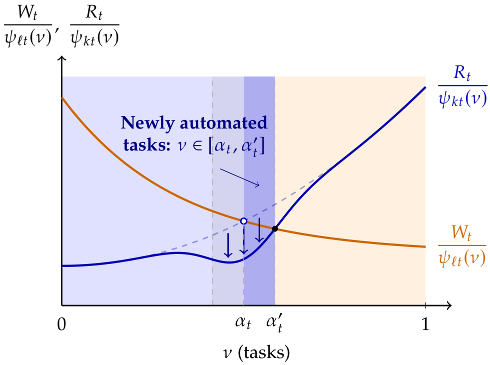


Figure 4. Technical Progress Near the Automation Cutoff.

Innovation Far Beyond Automation Cutoff

- Finally consider increases in $\psi_{kt}(\nu)$ well to the right of crossing point
- No immediate effect on economy
- No automation, no change in task prices
- But can build toward eventual automation

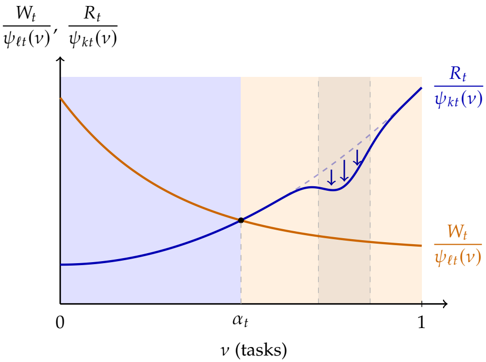


Figure 5. Technical Progress Far Beyond the Automation Cutoff.

SIMPLE CASE (UNIFORM PRODUCTIVITIES)

- We assume:

$$\psi_{kt}(\nu) = \begin{cases} \psi_{kt} & \text{for} \quad \nu \in [0, \alpha_t] \\ 0 & \text{for} \quad \nu \in [\alpha_t, 1] \end{cases}$$

$$\psi_{\ell t}(\nu) = \quad \psi_{\ell t} \quad \text{for} \quad \nu \in [0, 1]$$

- Allows us to study changes in α_t and ψ_{kt} separately
- These capture pure productivity and automation effects

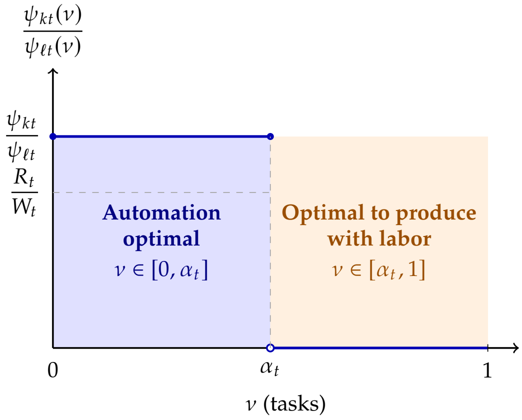


Figure 6. Uniform Productivities.

Simple Case (Uniform Productivities)

- Using capital is only profitable if $\frac{R_t}{\psi_{kt}} \leq \frac{W_t}{\psi_{\ell t}}$
  So, this condition must hold
- If economy has very little capital: $\frac{R_t}{\psi_{kt}} = \frac{W_t}{\psi_{\ell t}}$
  (labor also used to produce tasks $[0, \alpha_t]$)
- With sufficient capital: $\frac{R_t}{\psi_{kt}} < \frac{W_t}{\psi_{\ell t}}$
  (Case drawn in figure)
- We focus on this last case

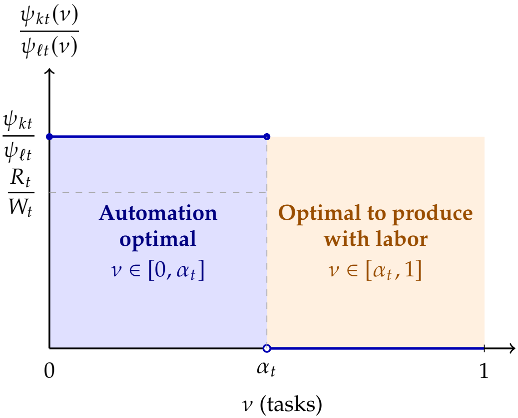


Figure 6. Uniform Productivities.

Labor Demand

- Task-specific labor demand for tasks $\nu \in [\alpha_t, 1]$: $L_t(\nu) = \frac{Y_t(\nu)}{\psi_{\ell t}}$
- Using demand for task ν: $L_t(\nu) = \frac{P_t(\nu)^{-\sigma} Y_t}{\psi_{\ell t}}$
- Using price of task ν: $L_t(\nu) = \psi_{\ell t}^{\sigma-1} W_t^{-\sigma} Y_t$
- So we have that task specific labor demand is

$$L_t(\nu) = \begin{cases} \psi_{\ell t}^{\sigma-1} W_t^{-\sigma} Y_t & \text{if } \nu \in [\alpha_t, 1] \\ 0 & \text{otherwise} \end{cases}.$$

Wage

- Suppose (for simplicity) that L_t labor is supplied inelastically
- Equilibrium in the labor market entails

$$L_t = \int_{\alpha_t}^{1} L_t(\nu) d\nu = (1 - \alpha_t)\, \psi_{\ell t}^{\sigma - 1} W_t^{-\sigma} Y_t$$

- Solving for W_t

$$W_t = (1 - \alpha_t)^{\frac{1}{\sigma}}\, \psi_{\ell t}^{\frac{\sigma - 1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}$$

- Very similar to earlier CES model, except that α_t can change

Capital and Rental Rate

- Suppose economy is endowed with a fixed amount of capital K_t
- Analogous step for capital yield

$$K_t(\nu) = \begin{cases} \psi_{kt}^{\sigma-1} R_t^{-\sigma} Y_t & \text{if } \nu \in [0, \alpha_t] \\ 0 & \text{otherwise} \end{cases} .$$

$$K_t = \int_0^{\alpha_t} K_t(\nu) d\nu = \alpha_t \psi_{kt}^{\sigma-1} R_t^{-\sigma} Y_t$$

$$R_t = \alpha_t^{\frac{1}{\sigma}} \psi_{kt}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}$$

Aggregate Production Function

- Interestingly, one can derive a CES aggregate production function for this model:

$$Y_t = \left[(A_{kt}K_t)^{\frac{\sigma-1}{\sigma}} + (A_{\ell t}L_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  where

$$A_{kt} = \alpha_t^{\frac{1}{\sigma-1}}\psi_{kt} \quad \text{and} \quad A_{\ell t} = (1-\alpha_t)^{\frac{1}{\sigma-1}}\psi_{\ell t}.$$

- Factor share are:

$$s_{kt} = \frac{R_t K_t}{Y_t} = \left(\frac{A_{kt}K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} \quad \text{and} \quad s_{\ell t} = \frac{W_t L_t}{Y_t} = \left(\frac{A_{\ell t}L_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}.$$

- See lecture notes for derivation

Interpretation of A_{kt} and $A_{\ell t}$

- A_{kt} and $A_{\ell t}$ traditionally referred to as capital and labor augmenting technical change
- But is a new more powerful computer:
  - Like having more old computers (A_{kt})
  - Allows worker to economize on time ($A_{\ell t}$)
- Task-based framework clarifies this
- First, new computer raises $\psi_{kt}(\nu)$ for some set of tasks (which raises A_{kt})
- Second, potentially also raises α_t
- How does this affect

$$A_{kt} = \alpha_t^{\frac{1}{\sigma-1}} \psi_{kt}$$
$$A_{\ell t} = (1-\alpha_t)^{\frac{1}{\sigma-1}} \psi_{\ell t}$$

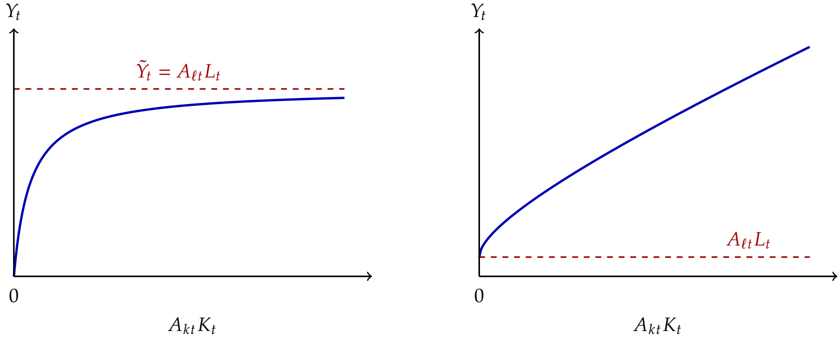
- Effect on α_t depends on whether σ is larger and smaller than 1
- If $\sigma < 1$, increase in α_t raises $A_{\ell t}$
- Computer neither purely capital augmenting nor purely labor augmenting

Weak Links

- Technical change can lead to breathtaking increase in productivity of particular tasks (e.g., cotton spinning in 18th century, computing in 20th century, AI/robotics in 21st century)
- How does this affect overall output?
- Depends critically on how substitutable different tasks are:

$$Y_t^{\frac{\sigma-1}{\sigma}} = (A_{kt}K_t)^{\frac{\sigma-1}{\sigma}} + (A_{\ell t}L_t)^{\frac{\sigma-1}{\sigma}}$$

- If $\sigma < 1$, when if $A_{kt}K_t \to \infty$, then $(A_{kt}K_t)^{\frac{\sigma-1}{\sigma}} \to 0$.
- In this limit, therefore $Y_t = A_{\ell t}L_t$



(a) Gross Complements ($\sigma < 1$)

(b) Gross Substitutes ($\sigma > 1$)

Figure 7. Aggregate Output as Capital Becomes Infinitely Productive.

- $A_{\ell t} L_t = 1$. Left panel has $\sigma = 0.5$. Right panel has $\sigma = 2$

Baumol's Cost Disease

- $(A_{kt}K_t)^{\frac{\sigma-1}{\sigma}} \to 0$ when $A_{kt}K_t \to \infty$ and $\sigma < 1$ is an instance of Baumol's cost disease
- Relative task demand:
$$\frac{Y_t(\nu)}{Y_t(\nu')} = \left[\frac{P_t(\nu)}{P_t(\nu')}\right]^{-\sigma}$$
- When $\sigma < 1$, a 1% fall in $P_t(\nu)/P_t(\nu')$ leads to a less than 1% increase in $Y_t(\nu)/Y_t(\nu')$
- This implies that relative spending on the task ν falls
- Tasks (or goods) with high productivity growth see their expenditure share wither away
- Tasks (or goods) with low productivity growth come to dominate the economy

BENEFITS FROM INFINITE PRODUCTIVITY IN CERTAIN TASKS

- How much does output rise if capital becomes infinitely productive?

$$Y_t^{\frac{\sigma-1}{\sigma}} = (A_{kt}K_t)^{\frac{\sigma-1}{\sigma}} + (A_{\ell t}L_t)^{\frac{\sigma-1}{\sigma}}$$

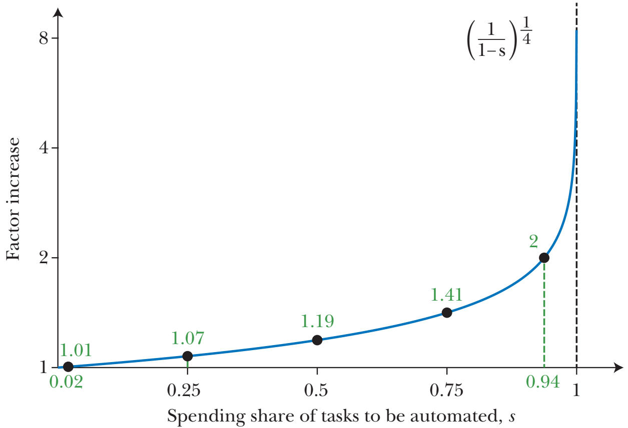
- Defining $\tilde{Y}_t = A_{\ell t}L_t$ and using $s_{kt} = \left(\frac{A_{kt}K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}$:

$$Y_t^{\frac{\sigma-1}{\sigma}} = s_{kt}Y_t^{\frac{\sigma-1}{\sigma}} + \tilde{Y}_t^{\frac{\sigma-1}{\sigma}}$$

- Rearranging:

$$\frac{\tilde{Y}_t}{Y_t} = \left(\frac{1}{1-s_{kt}}\right)^{\frac{\sigma}{1-\sigma}}$$

Benefits from Infinite Productivity in Certain Tasks



- $\frac{\tilde{Y}_t}{Y_t} = \left(\frac{1}{1-s_{kt}}\right)^{\frac{\sigma}{1-\sigma}}$ as a function of s_{kt} for $\sigma = 0.2$. Source: Jones (2026).

- Log-differentiating the aggregate production function yields

$$\frac{d \ln Y_t}{dt} = s_{kt}\left(\frac{d \ln A_{kt}}{dt} + \frac{d \ln K_t}{dt}\right) + s_{\ell t}\left(\frac{d \ln A_{\ell t}}{dt} + \frac{d \ln L_t}{dt}\right)$$

- Rearranging yields Solow (1957) residual:

$$\frac{d \ln Y_t}{dt} - s_{kt}\frac{d \ln K_t}{dt} - s_{\ell t}\frac{d \ln L_t}{dt} = s_{kt}\frac{d \ln A_{kt}}{dt} + s_{\ell t}\frac{d \ln A_{\ell t}}{dt}$$

or written more compactly

$$\frac{d \ln TFP_t}{dt} = s_{kt}\frac{d \ln A_{kt}}{dt} + s_{\ell t}\frac{d \ln A_{\ell t}}{dt}$$

Technical Change and Productivity

- Recall that

$$A_{kt} = \alpha_t^{\frac{1}{\sigma-1}} \psi_{kt} \quad \text{and} \quad A_{\ell t} = (1-\alpha_t)^{\frac{1}{\sigma-1}} \psi_{\ell t}$$

- Log-differentiating these yields

$$\frac{d \ln A_{kt}}{dt} = \frac{1}{\sigma - 1} \frac{d \ln \alpha_t}{dt} + \frac{d \ln \psi_{kt}}{dt}$$

$$\frac{d \ln A_{\ell t}}{dt} = -\frac{1}{\sigma - 1} \frac{\alpha_t}{1-\alpha_t} \frac{d \ln \alpha_t}{dt} + \frac{d \ln \psi_{\ell t}}{dt}$$

- Plugging this back into expression for $d \ln TFP_t / dt$ yields

$$\frac{d \ln TFP_t}{dt} = s_{kt} \frac{d \ln \psi_{kt}}{dt} + s_{\ell t} \frac{d \ln \psi_{\ell t}}{dt} + \frac{1}{\sigma - 1} \left(s_{kt} - \frac{\alpha_t}{1-\alpha_t} s_{\ell t} \right) \frac{d \ln \alpha_t}{dt}$$

- Recall that

$$K_t = \alpha_t \psi_{kt}^{\sigma-1} R_t^{-\sigma} Y_t \quad \text{and} \quad L_t = (1-\alpha_t)\,\psi_{\ell t}^{\sigma-1} W_t^{-\sigma} Y_t$$

- This implies

$$s_{kt} = \alpha_t \left(\frac{R_t}{\psi_{kt}}\right)^{1-\sigma} \quad \text{and} \quad s_{\ell t} = (1-\alpha_t)\left(\frac{W_t}{\psi_{\ell t}}\right)^{1-\sigma}.$$

- Which implies that

$$s_{kt} - \frac{\alpha_t}{1-\alpha_t} s_{\ell t} = \alpha_t \left[\left(\frac{R_t}{\psi_{kt}}\right)^{1-\sigma} - \left(\frac{W_t}{\psi_{\ell t}}\right)^{1-\sigma}\right].$$

Technical Change and Productivity

- Plugging this last expression back into the expression for $d \ln TFP_t / dt$ yields

$$\frac{d \ln TFP_t}{dt} = \underbrace{s_{\ell t} \frac{d \ln \psi_{\ell t}}{dt}}_{\substack{\text{Growth in Productivity} \\ \text{of Labor}}} + \underbrace{s_{kt} \frac{d \ln \psi_{kt}}{dt}}_{\substack{\text{Growth in Productivity} \\ \text{of Capital}}} + \underbrace{\frac{1}{\sigma - 1} \left[\left(\frac{R_t}{\psi_{kt}} \right)^{1-\sigma} - \left(\frac{W_t}{\psi_{\ell t}} \right)^{1-\sigma} \right] \frac{d\alpha_t}{dt}}_{\text{Automation}}$$

- Three sources of productivity gains

- Jones and Tonetti (2026) argue that most TFP growth comes from $d \ln \psi_{kt} / dt$

$$\frac{d \ln TFP_t}{dt} = s_{\ell t}\frac{d \ln \psi_{\ell t}}{dt} + s_{kt}\frac{d \ln \psi_{kt}}{dt} + \underbrace{\frac{1}{\sigma - 1}\left[\left(\frac{R_t}{\psi_{kt}}\right)^{1-\sigma} - \left(\frac{W_t}{\psi_{\ell t}}\right)^{1-\sigma}\right]\frac{d\alpha_t}{dt}}_{\text{Automation}}$$

- Automation term is zero if $\frac{R_t}{\psi_{kt}} = \frac{W_t}{\psi_{\ell t}}$ (smooth automation case)
- But this does not mean that automation doesn't matter for productivity
- Once goods are automated, they start benefiting from $d \ln \psi_{kt}/dt$
- Without continual automation, capital share (and TFP growth) would wither

Technical Change and Wages

- Recall that

$$W_t = (1-\alpha_t)^{\frac{1}{\sigma}} \psi_{\ell t}^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}}$$

- This implies that

$$\frac{d \ln W_t}{d \ln \psi_{kt}} = \frac{1}{\sigma} \frac{d \ln Y_t}{d \ln \psi_{kt}} = \frac{s_{kt}}{\sigma}$$

  where we use the fact that the aggregate production function implies that $\frac{d \ln Y_t}{d \ln \psi_{kt}} = s_{kt}$

- An increase in ψ_{kt} unambiguously increases wages (pure productivity effect)

- Similar derivation yields: $\frac{d \ln W_t}{d \ln K_t} = \frac{s_{kt}}{\sigma}$, a pure “scarcity” effect

Technical Change and Wages

- Suppose instead that the productivity of labor increases:

$$\frac{d\ln W_t}{d\ln \psi_{\ell t}} = \frac{1}{\sigma}\left[\frac{d\ln Y_t}{d\ln \psi_{\ell t}} - (1-\sigma)\right] = \frac{\sigma - s_{kt}}{\sigma}$$

- Effect ambiguous
- "Direct" productivity effect (positive)
- "Abundance" effect (negative)
  - Increase in $\psi_{\ell t}$ makes labor (effectively) more abundant (increases $\psi_{\ell t} L_t$)
  - This pushes down price of labor
- Increase in L_t yields a pure abundance effect: $\frac{d\ln W_t}{d\ln L_t} = -\frac{s_{kt}}{\sigma}$

Technical Change and Wages

- Finally, consider automation:

$$\frac{d \ln W_t}{d \ln \alpha_t} = \frac{1}{\sigma}\left(\frac{d \ln Y_t}{d \ln \alpha_t} - \frac{\alpha_t}{1-\alpha_t}\right)$$

- Overall effect ambiguous
- Productivity effect: $\frac{1}{\sigma}\frac{d \ln Y_t}{d \ln \alpha_t} \geq 0$ (zero if $\frac{R_t}{\psi_{kt}} = \frac{W_t}{\psi_{\ell t}}$)
- Displacement effect: $-\frac{1}{\sigma}\frac{\alpha_t}{1-\alpha_t} < 0$
- Displacement effect is new relative to traditional production models
- Capital takes over tasks. This reduces labor demand.
- As a consequence: Better machines can hurt workers

Technical Change and the Labor Share

- Derivations similar to those done above yield

$$\frac{d \ln s_{\ell t}}{d \ln \alpha_t} = -\left(\frac{s_{kt}}{\sigma}\right)\left(\frac{1}{1-\alpha_t}\right) < 0.$$

- Automation unambiguously reduces the labor share

$$\frac{d \ln s_{\ell t}}{d \ln \psi_{kt}} = -\frac{d \ln s_{\ell t}}{d \ln \psi_{\ell t}} = \frac{d \ln s_{\ell t}}{d \ln (K_t/L_t)} = s_{kt}\left(\frac{1-\sigma}{\sigma}\right)$$

- If $\sigma < 1$, increases in ψ_{kt} raise the labor share (Baumol's disease)

Uzawa (1961) Theorem in Task Framework

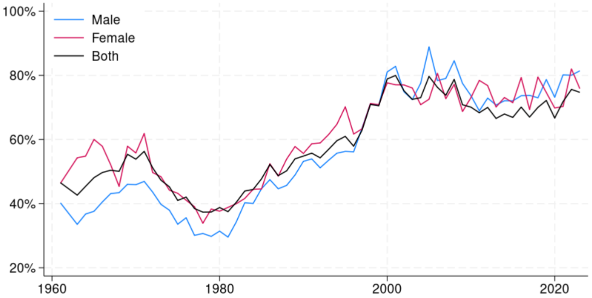
- Recall that Uzawa's (1961) theorem says that on a balanced growth path all technical progress must be labor augmenting (i.e., A_{kt} must be constant in the long run)
- In our task model: $A_{kt} = \alpha_t^{\frac{1}{\sigma-1}} \psi_{kt}$
- So we have that
$$\frac{d \ln A_{kt}}{dt} = \frac{1}{\sigma - 1} \frac{d \ln \alpha_t}{dt} + \frac{d \ln \psi_{kt}}{dt}$$
- Jones and Liu (2024):
  - If $\sigma < 1$, technical progress that increases both α_t and ψ_{kt} in the right proportions can leave A_{kt} unchanged
  - All technical progress can be capital improving at the micro level, but purely "labor augmenting" at the macro level

Appendix

Skill Premium

- The College-High School wage premium rose substantially between 1980 and 2000

Figure 1: College to high school earnings gap, overall and by gender



Note: Years are survey reference years. Earnings are weekly earnings, measured as defined in the text. The sample used to compute earnings includes only full-time wage and salary workers aged 25–64 with exactly a college or high school degree who earn at least $50 per week (in 1989 dollars).
Source: Authors' calculations from CPS ASEC microdata.

Back

(Bengali, Valletta, and Zhao (2025))